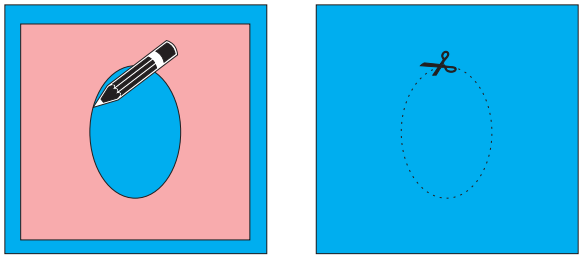
c. Volume of the Sphere, Ellipsoid and Ovoid

Material:

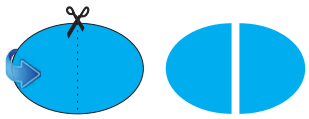
- Plane insets for the circle, ellipse, and oval from the Geometry Cabinet
- Circle halves from the fractional insets
- Envelope
- Sphere, ellipsoid, ovoid solids

Presentation One:

1. Take blue paper from envelope and the frame of the oval. Draw the outline of the oval and cut out.

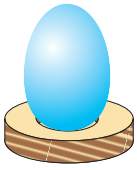


2. Fold the oval in half and cut it to form 2 halves.



3. Take the semi-oval and rotate it to imagine forming the solid. We form a figure that looks like an egg.

4. Place the solid ovoid in the oval frame.



5. Repeat with the ellipse.
6. Take the half circle from the fraction insets and repeat the above steps.

Material:

- Sphere
- Cardboard rectangle to be constructed into a cylinder equal in circumference to the sphere and equal in height to the sphere. The sphere must fit into this cylinder snugly.
- Sand, scoop, overflow tray, funnel
- Pie tin, ruler
- Cylinder from volume solids
- Paper
- Pencil
- Labels

Presentation Two:

1. We want to find the volume of the sphere, but we need a mediator - the cylinder.
2. Recall the formula for the volume of the cylinder. Set is out in the labels:

   $V = \pi r^2 h$
3. Slide the sphere into the cylinder. Check that the diameter and the height of both is the same.
4. We can rewrite the formula for the volume of this particular cylinder this way, replacing the "h" label with the "d" label:

   $V = \pi r^2 d$

   This is the formula for this particular cylinder only.
5. We can change the formula further. Since the diameter equals twice the radius, remove the "d" label and substitute "2" and "r" for it. We now have:

   $V = \pi r^2 d \quad d = 2r$

   $V = \pi r^2 \times 2r$

   $V = 2 \pi r^3$

6. Now we need to find out how much of the volume of the cylinder is occupied by the sphere inside it. We will use sand, as we did for the cone.
    a. Place the cylinder in the overflow tray.
    b. Fill up the space above the sphere with sand, leveling it off with a ruler. Let the overflow accumulate on the tray.
    c. Carefully pour off sand from the cylinder into the pie tin. This sand represents the volume of the space above the sphere.
    d. Turn over the cylinder and sphere. Place them in the overflow tray. Add sand. Level it with a ruler.
    e. Add this sand to that in the pie tin.
    f. The sand in the tin represents the space in the cylinder not occupied by the sphere.
7. Remove the sphere from inside the cylinder. With the help of a funnel, pour back the sand from the pie tin into the cylinder. Estimate what fraction of the cylinder is filled with sand (about one third). This means that the sphere occupied two thirds of the space inside the cylinder.
8. Return to the formula for the volume of the cylinder. The volume of the sphere is two thirds of the volume of the cylinder.

$$V = 2/3 \times 2 \pi r^3$$

$$V = 4/3 \pi r^3$$

D. Volume of Regular Polyhedrons

Material:

- 5 regular polyhedrons:
- tetrahedron
- hexahedron
- octahedron
- dodecahedron
- isocahedron
- Triangular inset (4 equilateral triangles)
- Black noun symbol
- Tiling game
- Paper plates, paper, pencil, scissors, tape
- Labels

Tetrahedron

Material:

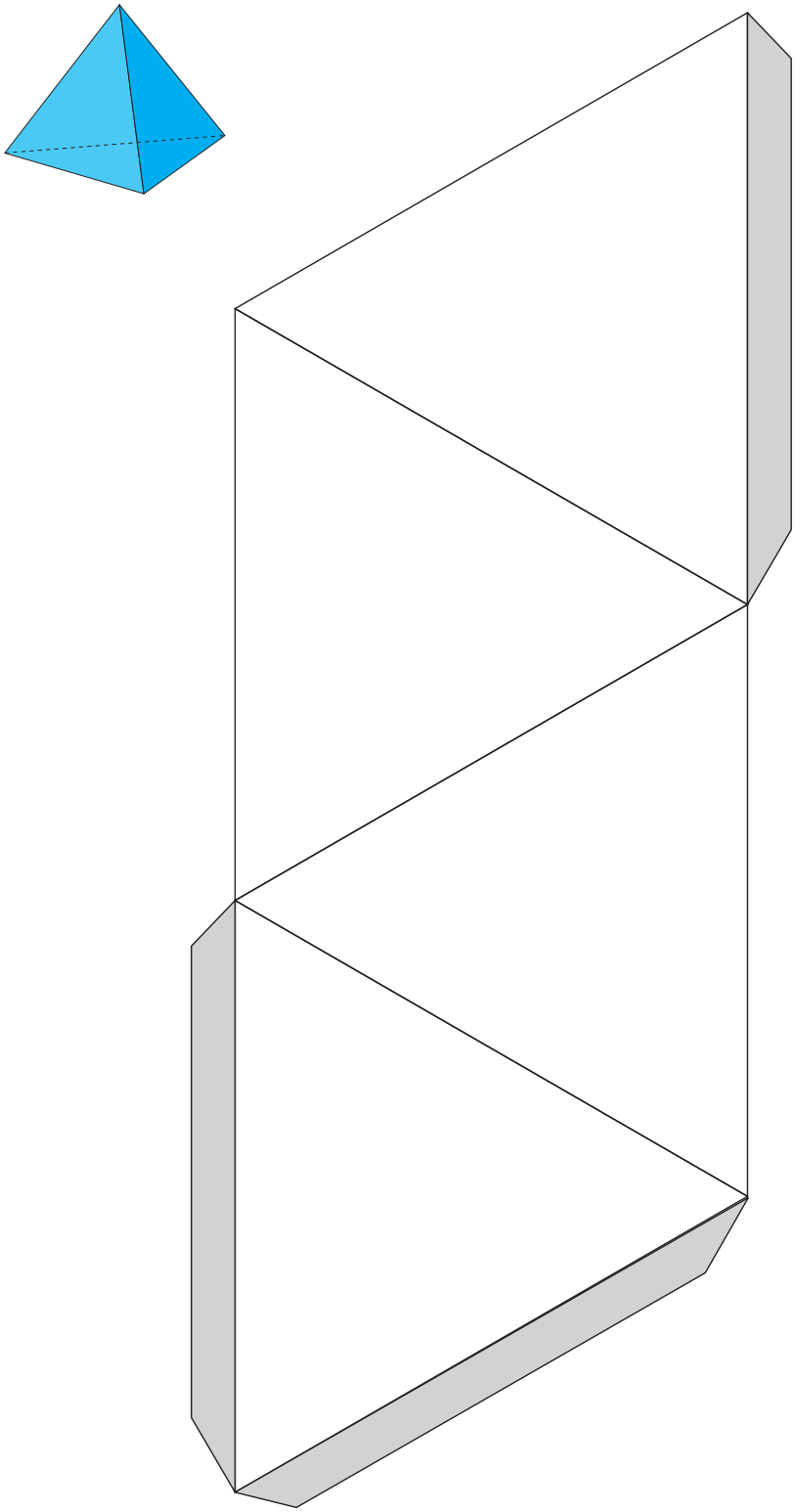
- 4 small red equilateral triangles from the Triangle box

Presentation:

1. Trace the 4 equilateral triangles on paper so that they form one large equilateral triangle. Fold the four triangles on their sides. Shape the triangle into a tetrahedron. Tape together. The number of faces, 4, gives the name. All the faces are equilateral triangles.

2. The triangle was the first figure in plane figures. The tetrahedron is the first figure we can build in space. The triangle was the "constructor", even of the circle. The tetrahedron is the builder of all the other solids.

Tetrahedron paper model



Cube: Hexahedron

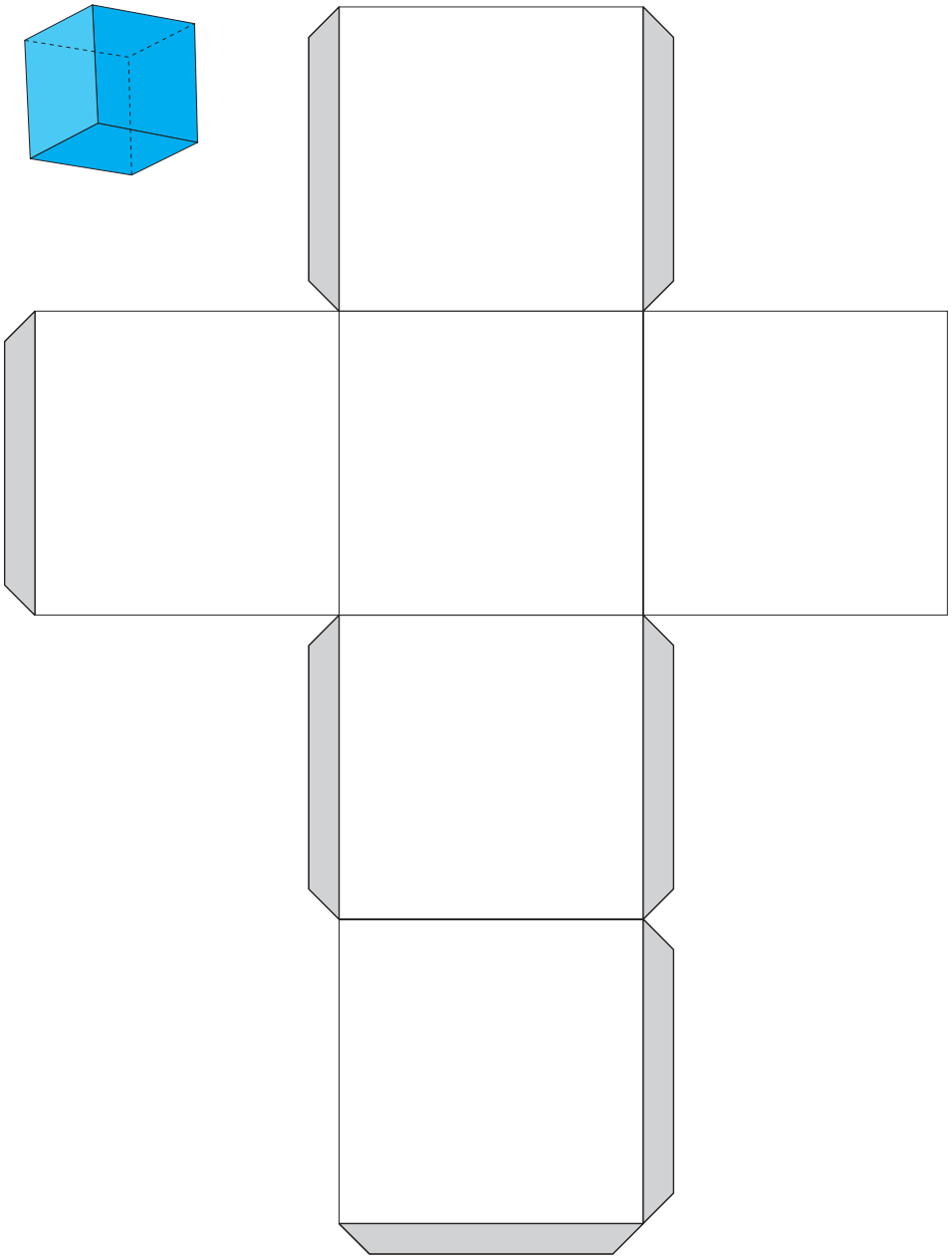
Material:

- All the cubes of the Pink Tower plus the blue cube from volume material

Presentation:

1. Trace one of the cubes on paper. Cut it out. Fold the paper into a cube. Tape. The number of faces, 6, gives the name. All the faces are squares.

Cube paper model



Octahedron

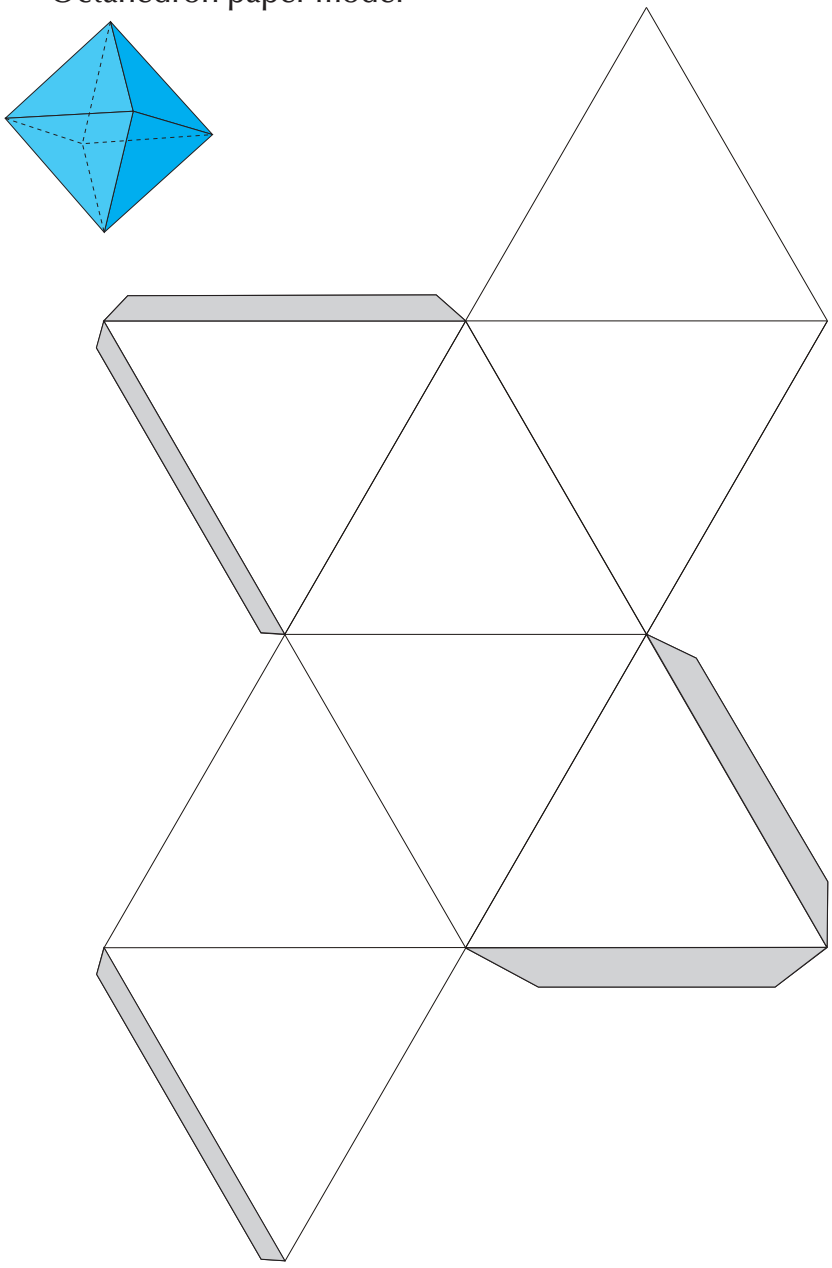
Material:

- Noun symbol of pyramid

Presentation:

1. If we double this pyramid, we have an octahedron. The number of faces, 8, gives the name. All the faces are equilateral triangles

Octahedron paper model



Dodecahedron

Material:

- Tiling box regular pentagons

Presentation:

1. Construct the dodecahedron from the regular pentagons. The number of faces, 12, gives the name. All the faces are regular pentagons.
2. Trace the pentagons on paper. Cut them out. Construct the dodecahedron and tape together.

Dodecahedron paper model