3. FROM THE SQUARE TO THE CUBE OF A NUMBER

Introduction:

Numbers can be compared to nations. There are both large and small nations just as there are large and small number quantities. Both nations and number quantities are guided by perfect laws and must obey the law of the 'group'.

In the exercises on squaring, the child becomes aware of the passage from a number to its square.

(Example: The bar of seven is made up of 7 units. The square of seven is made up of 7 bars of seven.)

This is the **Law of the Group,** which dominates the 'Nation of Seven'. If this rules the formation of the Square, it will also apply to the formation of the Cube (seven squares to build it).

Material:

- Colored Squares and Cubes

Presentation:

1. Explain that the **cube of seven** means $7^2 \times 7$, which is demonstrated by replacing the 7 squares of 7 with the cube of 7.
2. Lay out the square of each colored bead bar from 1 to 10.
3. Stack the corresponding number of squares to form the cube of that number, showing a progression in size and height. Label each set of cubes:

$1^2 \times 1$	$2^2 \times 2$

4. Replace the stacked squares with real cubes. Turn over the labels of each cube and rewrite the label:

1^3	2^3	3^3	etc.

5. Emphasize that just as 2^2 (two to the 2nd power) is always a square, 2^3 is always a cube. When a number is multiplied by itself, it is written two times (2 x 2). When a number is multiplied by itself three times, it is written out three times (2 x 2 x 2), etc. 2^5 means 2 multiplied by itself 5 times (2 x 2 x 2 x 2 x 2). It is essential that the child understands exponents.

1^2 x 1	**2^2 x 2**	**3^3 x 3**	**4^2 x 4**	**5^2 x 5**	**6^2 x 6**	**7^2 x 7**	**8^2 x 8**	**9^2 x 9**	**10^2 x 10**
1^3	**2^3**	**3^3**	**4^3**	**5^3**	**6^3**	**7^3**	**8^3**	**9^3**	**10^3**

4. FROM A CUBE TO A SUCCEEDING CUBE

Material:

- Large Tray of the Cube Root containing prisms of the 'Square' and of the 'Cube' of each number from 1 to 9, with colors corresponding to the Bead Bars

The cube of each number is a darker shade than the squares.

Presentation 1: First Passage

Example: Moving from the cube of 4 to the cube of 5

1. Place the wooden cube of 4 in front of the cube of 5. Then find out the 3 dimensions which are the 3 faces of the cube. A larger cube is formed by 'growing' in three directions.
2. To do this, position 3 yellow wooden squares of 4 on the 3 faces of the cube of 4.
3. Next, fill out the 3 edges with red or neutral cubes such that each of the 3 bars is made up of 4 x 1. Complete the figure with 1 cube at the vertex (1^3).

Presentation 2: Second Passage - Writing

1. Write out the above.

cube, 3 faces, 3 edges, vertex

$$4^3 + \underbrace{4^2 + 4^2 + 4^2}_{\text{3 faces}} + \underbrace{(4 \times 1) + (4 \times 1) + (4 \times 1)}_{\text{3 edges}} + 1^3 = \text{Cube of 5}$$

$$64 + 16 + 16 + 16 + 4 + 4 + 4 + 1 = 125$$

2. Both cubes now have the total of 125 (cube of 5). Take away what was added and total it to find out the **difference** between the 2 cubes.

3 faces, each 4^2	48
3 edges, each 4	12
1 vertex	1
	61

61 represents the **difference** between the cubes

Presentation 3: Third Passage

Example: Moving from the cube of 8 to the cube of 9

1. Now the steps are written out immediately as the work is done.

cube faces edges vertex

$8^3 + \underbrace{8^2 + 8^2 + 8^2}_{\text{faces}} + \underbrace{(8 \times 1) + (8 \times 1) + (8 \times 1)}_{\text{edges}} + 1^3 = \text{cube of } 9$

$512 + 3(64) + 3(8) + 1 = 729 = 9^3$

Point of Consciousness:

Point out that this material represents the bead bars in a more ‘functional’ manner, and for this reason, the two materials may be correlated doing the same example.

5. FROM A CUBE TO A NON- SUCCEEDING CUBE

Material:

- Large Tray of the Cube Root containing prisms of the 'Square' and of the 'Cube' of each number from 1 to 9, with colors corresponding to the Bead Bars.

The cube of each number is a darker shade than the squares.

Presentation 1: First Passage - Sensorial

Example: Moving from the cube of 4 to the cube of 7

1. The objective is to find the difference between the cube of 7 and the cube of 4.
2. Line up the two different cubes, and find the three measurements or **faces** with yellow squares of 4.
3. Since the difference between the two cubes is 3 in every dimension, form the **faces** by taking the square of 4, three times for each face, thus making each face three layers deep.
4. Then the 'bars' (see Presentation 4, First Passage) are represented by the difference of three; so we have 4 layers of the square of 3, three distinct times.
5. The 'vertex' (see Presentation 3, First Passage) is completed with the cube of 3.

Presentation 2: Second Passage - Written

1. Calculate the volume of the cube.

$4^3 + (3 \times 4^2) + (3 \times 4^2) + (3 \times 4^2) + (3^2 \times 4) + (3^2 \times 4) + (3^2 \times 4) + 3^3 =$

$64 + 48 + 48 + 48 + 36 + 36 + 36 + 27 = 7^3$

Cube of 7 $7 \times 7 \times 7 = 343$

Cube of 4 $4 \times 4 \times 4 = \underline{64}$

279 **Difference** between the two cubes is the amount that was added to the cube of 4.